Market Implied Valuation of Cashflow CLO Structures

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Visual Summary of the Talk

Bridging the gap between CDO and CLO valuation: The Pessimistic Views

Visual Summary of the Talk

Bridging the gap between CDO and CLO valuation: The Optimistic View

Market Implied Valuation of Cashflow CLO Structures

Structure of the presentation

Motivation

The market context in the aftermath of the credit crunch

What is the value of a CLO tranche?

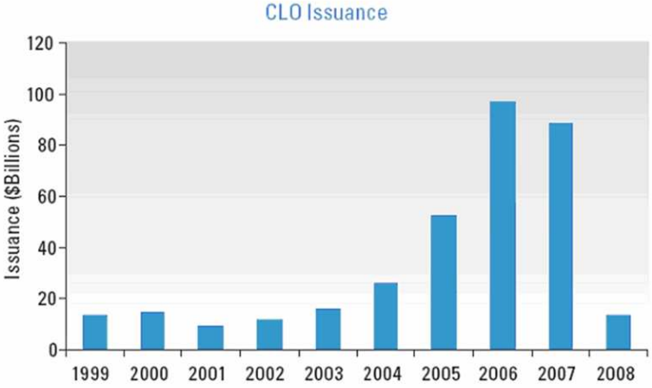
Complex and customized payoffs
Dynamic Leverage
One-of-a-kind portfolios
Embedded optionality
...

Is the question still relevant?

Issuance has dropped dramatically
"Legacy assets" will be around for some time

Motivation

Evolution of CLO issuance (US data, up to 2008)



Source: S&P Leveraged Commentary & Data, January 2009

1. Description of the CLO structure

The nature of the CLO market

Collateralized Loan Obligations

- A primary (origination) market for debt securities that repackage bank loans (leveraged)
- SPV required. Large number of involved parties to each transaction.
- Key roles for asset managers and rating agencies
- Closing a transaction requires simultaneous investors with different risk appetite (senior, mezzanine, equity)
- Structure described in 200-300 page document (indenture)
- Buy and hold product with limited secondary trading

1. Description of the CLO structure

Summary of the description

- The balance sheet
- Activities during each period
- Status determination (OC Test)
- Curing failed tests
- Waterfall of payments

Assets

Senior Secured Bank Loans.
Subordinated Loans.
Hight Yield Bonds
CLO mezzanine bonds
Other ...

Liabilities

Senior Management Fees
Senior Notes (AAA)
Other Senior Notes (AA, A)
Mezzanine Notes (B, BB, BBB)
Mezzanine Management Fees
"Equity"
Combination Notes

1. Description of the CLO structure

What happens during each period

- Collections of interest from assets
- Defaults/recoveries from assets
- Investments in new assets
- At payment date: determination of transaction "status"
- At payment date: payments of interest to notes, fees and equity according to status/waterfall
- Possible liquidation

The *supported debt level* for a bond is the sum of total bond sizes down to and including the m-th bond:

$$\sigma_k^m = \sum_{j=1}^{m} B_k^j. \tag{1}$$

The *overcollateralization* (OC) for the various bonds is then defined as:

$$L_k^m = \frac{N_k + r_k}{\sigma_k^m}. \tag{2}$$

The numerator is an *adjusted* outstanding notional to properly reflect the current leverage.

The running leverage and debt service ratios (L_k^m, I_k^m) are checked against trigger levels (L_B^m, I_B^m). The joint outcome affects the waterfall:

$$J_k^m = 1_{\{L_k^m > L_B^m, I_k^m > I_B^m\}}$$

Each of the test is either PASS or FAIL

The "supportable" notional size Q_k^m is:

$$Q_k^m = \frac{\tilde{N}_k}{L_B^m}$$

The *scheduled notional reduction* of the l-th bond to cure the m-th OC test:

$$\delta T_k^{lm} = \max(\min(\sigma_k^m - \sum_{j=1}^{l-1} \delta T_k^{jm} - Q_k^m, B_k^l), 0).$$

Successful cure is indicated by:

$$C_k^m = 1_{\{\sum_{j=1}^m \delta T_k^{jm} = 0\}}$$

1. Description of the CLO structure

The payment operator

$$\{F_k^{sen}, I_k, f_k^{sen}\} := \mathcal{P}(F_k^{sen}, I_k, f_k^{sen})$$

$$\begin{aligned} f_k^{sen} &= \min(F_k^{sen}, I_k) \\ I_k &:= \max(0, I_k - F_k^{sen}) \\ F_k^{sen} &:= F_k^{sen} - f_k^{sen}, \end{aligned}$$

(The symbol $:=$ means an update of an existing variable)

- *Payment of senior fees:* Use the available interest (and principal) proceeds account I_k to pay the senior fees F_k^{sen}.

$$\{F_k^{sen}, I_k, f_k^{sen}\} := \mathcal{P}(F_k^{sen}, I_k, f_k^{sen})$$

- *Payment of interest to senior bonds:* Use any remaining interest proceeds I_k to pay scheduled interest to senior notes.

$$\begin{aligned} \{S_k^1, I_k, b_k^1\} &:= \mathcal{P}(S_k^1, I_k, b_k^1) \\ B_k^1 &:= B_k^1 + S_k^1 \end{aligned}$$

Mezzanine bond interest payments loop: For each OC/IC test $m = 1, \ldots, M-1$.

Check the m-th joint indicator J_k^m:

- If $J_k^m = 1$, pay scheduled interest to the $m+1$ notes from the interest proceeds I_k. (So e.g., the Class B notes will receive payment if the Class A OC/IC test is PASS).
- If $J_k^m = 0$, we enter into OC/IC "cure mode".
  - If $C_k^m = 1$, the required notional reduction has been achieved. Now the waterfall reverts back to the Mezzanine loop.
  - If $C_k^m = 0$, the required reduction was not successful. Defer interest on notes from $(m+1)$ onwards, equity receives no dividend.

1. Description of the CLO structure

Sketch of priority of payments (Waterfall): Part 2, Cure mode

Use interest proceeds I_k, to sequentially amortize principal on notes down to the m-th note.

Meet any shortfall using principal proceeds P_k or the reserve account a_k.

In sequence for each of the notes $j = 1, \ldots, m$ perform the updates:

$$\begin{aligned} \{\delta T_k^{jm}, I_k, \widetilde{b}_k^j\} &:= \mathcal{P}(\delta T_k^{jm}, I_k, \widetilde{b}_k^j) \\ b_k^j &:= b_k^j + \widetilde{b}_k^j \\ B_k^j &:= B_k^j - \widetilde{b}_k^j \end{aligned}$$

Sketch of priority of payments (Waterfall): Part 3, Equity payments

- *Payment of mezzanine fees:* Use the available interest proceeds account I_k to pay the mezzanine fees F_k^{mez}.
- *Payment of Equity dividends:* If junior most OC/IC test J_k^M
- If $J_k^M = 1$, make equity payments.
- If $J_k^M = 0$, sequentially amortize *all* the notes, until the test is cured. Check whether the M-th (junior) OC/IC cure has been successful.
  - If $C_k^M = 1$, the required notional reduction has been achieved.
  - If $C_k^M = 0$, the required reduction was not successful. Equity receives no payment this period.

1. Description of the CLO structure

CDO vs CLO: More than one letter of difference

CLO: Cash Security

Complete capital structure
Equity receives residual cash
Tranche does not reduce on loss
Early amortization triggers
Embedded options

CDO: Synthetic Derivative

Single Tranche
Bilateral Contract
Equity receives fixed spread
Tranche notional reduces on default/losses

Prioritization of loses is the main ingredient of a synthetic structure

$$\mathrm{TL}_k^m = \min(U^m - L^m, \max(L_k - L^m, 0))$$

2. Existing valuation approaches

Equivalent rating method

Infer a *credit rating* for a CLO note. Use the rating to compare with observed valuations of other similarly rated notes.

- Credit rating assignment step is typically done using simulations
- Use credit ratings, loss characteristics of portfolio assets, along with default correlations. Quantitative study under historical measure.
- The tools and assumptions used are controlled by credit rating agencies
- The simulation and credit rating is thus obtained under some built-in historical measure
- Identify market price for tranches which, under similar analysis, are shown to exhibit obtain same rating

2. Existing valuation approaches

Equivalent rating method

Issues and weaknesses of the equivalent rating method

- Estimating portfolio losses under the historical measure is far from trivial due to scarcity of data.
- Correlation estimates are generally held fixed in time. Valuation changes over time cannot be linked to correlation changes.
- Does not provide valuations for the non-rated elements of the structure (management fees, equity)
- Questionable when structural differences between tranches cannot be captured by one-dimensional rating measure
- Vulnerable to loss of confidence in ratings

2. Existing valuation approaches

Asset & Liability Method

Compute liability cashflows directly

- Asset cashflows computed using simulations (historical measure)
- Liability cashflows are estimated by processing the portfolio cashflows through the indenture (the CLO contract)
- Compute present value using an appropriate "risky" discount factor

Pros & Cons

- More adapted to valuation than rating method
- Problematic to identify discount factors
- Inconsistent treatment of correlation

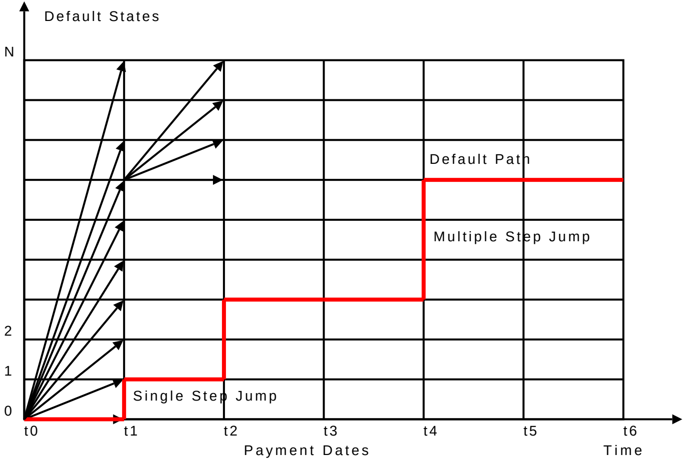
3. Elements of a market based approach

The conceptual framework

- Start with market prices: Assume a liquid traded market for tranched credit risk
- Derive expected tranche losses: Per period and subordination (attachment)
- Derive loss probabilities: To exceed given loss levels at different times (multiple solutions)
- Infer loss/default paths: Fit model to market implied marginal probabilities (multiple solutions)
- Valuation: Compute CLO payoffs per path and average

3. Elements of a market based approach

General top-down portfolio default model with multiple jumps



The default process will be developing according to the equation

$$P(D_k = \delta_i) = \sum_{j=0}^{i} P(D_{k-1} = \delta_j) P(D_k = \delta_i | D_{k-1} = \delta_j)$$

or in more concise notation

$$p_k^i = \sum_{j=0}^{i} p_{k-1}^j T_{k,k-1}^{ji}$$

- We assume the loss process is just a scalar multiple of the default process
- The loss discretization is not directly linked to the number of assets

3. Elements of a market based approach

Summary of Calculation

- Calculate default paths D_k (Markov Chain Simulation)
- Calculate loss L_k, notional N_k, interest and principal payments I_k, P_k
- For each payment date
  - compute current leverage ratios, check against triggers
  - calculate actual bond, fee and equity payments
  - update deferred bond notionals
- Average and discount payments with risk free rate

3. Elements of a market based approach

Discounting the calculated payoffs

The value of bonds, fees and equity...

$$\mathrm{PV}(B^m) = \sum_{k=1}^{n} E[b_k^m] \tag{3}$$

$$\mathrm{PV}(f^{sen}) = \sum_{k=1}^{n} E[f_k^{sen}] \tag{4}$$

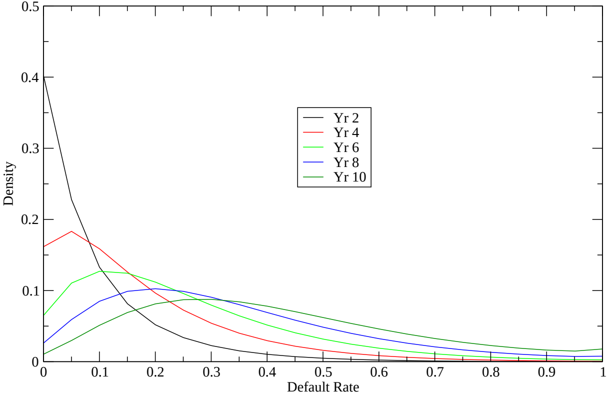
$$\mathrm{PV}(f^{mez}) = \sum_{k=1}^{n} E[f_k^{mez}] \tag{5}$$

$$\mathrm{PV}(E) = \sum_{k=1}^{n} E[d_k] \tag{6}$$

does not add up to the value of the assets. Embedded options

4. Examples of calculations

The evolution of the probability distribution of the default rate



4. Examples of calculations

Parameters of the synthetic structure

Tranche Parameters				Zero Loss Cashflows		
Name	*Lower Bound*	*Upper Bound*	*Spread*	*Per Annum*	*Total Coupon*	*Total*
Equity	0.00	0.15	0.1	0.015	0.15	0.3
Mezzanine	0.15	0.30	0.01	0.0015	0.015	0.165
Senior	0.30	1.00	0.005	0.0035	0.035	0.735
Sum				0.02	0.2	1.2

- For comparison purposes we think of the tranche in credit linked note format (principal exchange)
- 10yr maturity, annual payments

4. Examples of calculations

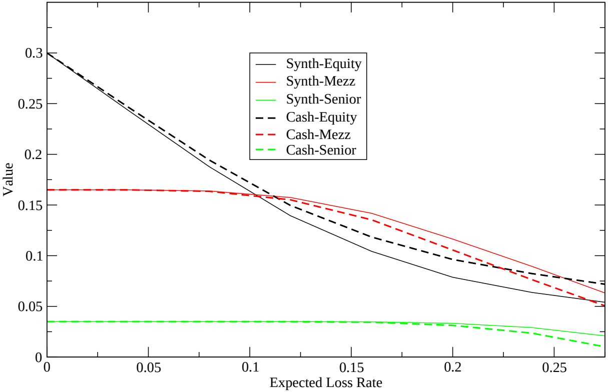
Parameters of the cashflow structure

Asset Parameters			Zero Loss Cashflows		
Portfolio	*Size*	*Spread*	*Per Annum*	*Total Coupon*	*Total*
Loans	1.0	0.02	0.02	0.2	1.2
Liability Parameters			**Zero Loss Cashflows**		
Name	*Size*	*Spread*	*Per Annum*	*Total Coupon*	*Total*
Equity	0.15	N/A	0.015	0.15	0.3
Mezzanine	0.15	0.01	0.0015	0.015	0.165
Senior	0.70	0.005	0.0035	0.035	0.735
Sum			0.02	0.2	1.2

- Simplified structure, no triggers
- No management fees

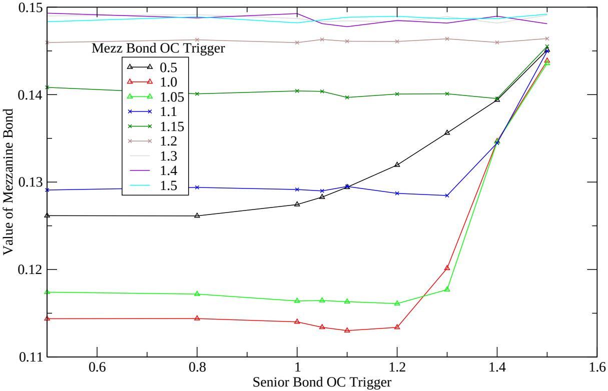
4. Examples of calculations

Illustration of the dependence of tranche values on the expected loss rate.



4. Examples of calculations

Illustration of the dependence of mezzanine note value on OC Trigger Levels



4. Examples of calculations

Value of liabilities as function of mezzanine fees

Mezz Fees p.a.	0.0000	0.0010	0.0020	0.0030	0.0040	0.0050	0.0060
Equity Value	0.1122	0.1046	0.0976	0.0918	0.0844	0.0782	0.0718
Mezzanine Value	0.1285	0.1282	0.1274	0.1264	0.1256	0.1253	0.1238
Senior Value	0.7254	0.7254	0.7253	0.7252	0.7251	0.7253	0.7250
Mezz Fees Value	0.0000	0.0076	0.0151	0.0226	0.0300	0.0374	0.0446
Fraction of par		75.9%	75.6%	75.4%	75.0%	74.8%	74.3%

- Equity affected strongly (first order). Senior notes insensitive
- Mezzanine notes and mezzanine fees(!) affected in a weak non-linear way

5. Open issues

Part 1: Incompleteness

- **Portfolio mapping** CLO portfolios will contain assets that are not part of an tradeable index.
- **Portfolio trading** Assumption that the CLO portfolio is fully ramped-up, static and maturity matched to the liabilities. Treatment of assets that default.
- **Haircuts and collateral quality tests** CCC bucket of a portfolio. Requires the forward population distribution of the underlying portfolio spreads.
- **Interest Rates** The CLO aims to largely hedge away any interest rate or exchange rate sensitivity. To the extend that the hedges employed to this effect are imperfect, the valuation will be influenced directly.

5. Open issues

Part 2: Embedded Optionality

- **Prepayment Option** Bank loans can typically be prepaid after a lock-out period. The prepayment incentive is primarily linked to the credit spreads.
- **Event-of-default Option** Where there is insufficient interest income to pay interest on the senior bonds) the structure may go into liquidation. The deciding option is held by senior bond holders
- **Equity Call Option** The equity investors have the option of calling the CLO after the so-called non-call period (optional redemption)

5. Open issues

Part 3: Unexpected(!) embedded optionality

Moody's says it did not foresee likelihood of KKR CLO tear-up

Monday, August 24, 2009

Moodys says it believes the recent developments affecting three KKR CLOs, in which **an affiliate of the manager tore up its mezzanine notes allowing the deals to come back into compliance with their overcollatearlisation tests**, are unlikely to herald a wave of similar actions in other CLOs....
KKRs moves are likely to have angered senior investors in the deals. As a result of the deals coming back into compliance with their par value tests, **cashflows that had been diverted to the senior notes will now be available to make payments to the equity and remaining mezzanine notes**.

Source: Creditflux

6. Conclusions

- Existing valuation techniques have serious shortcomings. Key inputs such as correlation are not transparently linked to market information
- Liquid benchmark portfolios (and tranche structures) can provide calibration information. Market standard model for implying a loss surface is required.
- Substantial remaining embedded options. Proposed framework is only a basis for an "option-adjusted-spread" methodology, not the final valuation