

Yann Brenier, Philippe Delanoë (UNS-CNRS) and Cédric Villani (ENS-Lyon) are organizing the following:

FRANCO-AUSTRALIAN (FAST) OPTIMAL TRANSPORTATION micro-WORKSHOP

at Laboratoire Dieudonné, NICE (France),
8-9 OCTOBER 2007

(partly supported by the Franco-Australian agreement **PHC FAST No.12739WA**)

List of participants:

E. Aubry, P. Bernard, M. Bernot, Y. Brenier, M. Castelpietra, P. Delanoë, L. De Pascale, A. Figalli, U. Frisch, Th. Gallouët, G. Lebeau, L. Rifford, S. Rigot, F. Robert, J. Roth, F. Santambrogio, N. Trudinger, C. Villani

Programme:

MONDAY OCTOBER 8, 2007

11h (main conference room) Neil **TRUDINGER** (CMA, ANU, Canberra, <http://www.maths.anu.edu.au/~neilt/>):

Title:

" SPICKS and SPECKS "

Abstract:

Conditions for diffeomorphism solutions of optimal transportation problems were found in recent years, in collaborations with Xi-nan Ma and Xu-jia Wang. In this talk, I report on some recent refinements of these results and the associated analysis of their convexity hypotheses.

14h30 (main conference room) Cedric **VILLANI** (ENS-Lyon, <http://www.umpa.ens-lyon.fr/~cvillani/>):

Title:

" Régularité du transport optimal en géométrie courbée : que sait-on ? "

Abstract:

Je ferai le point sur les avancées récentes dues à Trudinger, Wang, Loeper et d'autres sur la régularité du transport optimal dans un cadre de géométrie riemannienne, en insistant sur la notion de c-courbure.

15h30 PAUSE

16h (main conference room) Philippe **DELANOE** (UNS-CNRS, <http://math1.unice.fr/~delphi/>)

Title:

"Regularity of optimal transportation maps on compact Riemannian manifolds with slowly varying positive curvature" (with Yuxin **GE**)

Abstract:

On the standard sphere, given a couple of smooth positive probability measures, Gregoire Loeper (2006) proved the existence of a smooth optimal transportation map pushing one measure to the other. In collaboration with Yuxin Ge (Univ. Paris 12, LAMA), answering a question of Neil Trudinger, we extend Loeper's result to Riemannian metrics with slowly varying positive curvature.

17h15 (main conference room) Alessio **FIGALLI** (UNS-CNRS, <http://cvgmt.sns.it/people/figalli/>)

Title:

" C^1 regularity of solutions of the Monge-Ampère equation for optimal transport in dimension two" (with Grégoire **LOEPER**)

Abstract:

We prove C^1 regularity of c -convex weak Alexandrov solutions of the Monge-Ampère equation in two dimension assuming only a bound from above on the Monge-Ampère measure. Our result hold true under a natural condition on the cost function, namely non-negative cost-sectional curvature, that was shown by Grégoire Loeper to be necessary for C^1 regularity.

TUESDAY OCTOBER 9, 2007

10h (main conference room) Yann **BRENIER** (UNS-CNRS, <http://math1.unice.fr/~brenier/>):

Title:

"A connection between fluid mechanics and optimal transport theory through the Angenent, Haker and Tannenbaum equations and the Boussinesq equations"

Abstract:

Few years ago, Angenent, Haker and Tannenbaum introduced a gradient descent for the computation of the optimal transport map in the Monge-Kantorovich problem with applications in medical imaging (SIMA 2003).

We show that their equations can be derived (globally in time) from the Navier-Stokes-Boussinesq (NSB) equations (in a suitable generalized version and 2 space dimensions) as the inertia of the fluid is neglected. We partly use some recent (parallel) results of Chae and Hou-Li on the 2D NSB (which share some features with

the 3D Navier-Stokes equations).

This way, we establish a new connection between classical fluid mechanics and optimal transport theory.

11h15 (main conference room) Marc **BERNOT** (ENS-Lyon, <http://www.umpa.ens-lyon.fr/~mbernot/>):

Title:

" Color histogram transport "

Abstract:

An important aspect of a color image is its global color appearance, i.e. its palette. We'll see that the color histogram (which is the counting measure of the number of pixels with given colors) gives a good brief description of this palette. We shall then see how optimal transport permits to carry diverse palette transformations such as color transfer or color equalization. It will also be of interest to compare this approach with other transport of measures strategies.

14h30 (main conference room) Filippo **SANTAMBROGIO** (Univ. Paris-Dauphine, <http://linuz.sns.it/~santambroeus/mainlt.htm/>):

Title:

" Régularité et non régularité dans les problèmes de transport branché "

Abstract:

On veut considérer des propriétés de régularité des réseaux optimaux lors de l'irrigation d'une mesure diffuse, et notamment l'existence des tangentes. Celle-ci est essentielle pour énoncer des lois d'angles, correspondantes aux branchements à 120° dans le problème de Steiner. En particulier on montre que, dans l'irrigation d'une mesure avec densité bornée inférieurement et supérieurement à partir d'une seule source, toute courbe composant le réseaux a une dérivée avec variation totale localement bornée et admet donc des directions limites en tout point. Une ODE peut être déduite comme condition nécessaire d'optimalité, en donnant lieu aux conditions d'angles bien connues. Par contre, on a bâti un contre-exemple (un réseau qui n'admet pas de tangente en un point), avec une mesure irriguée singulière.

15h30 PAUSE

16h (main conference room) Luigi **DE PASCALE** (Univ. Pisa, <http://www.dm.unipi.it/~depascale/>):

Title:

" Local solutions and existence of optimal transport maps for the W_∞ Wasserstein distance. Applications to more classical Monge problems " (with T. **CHAMPION** & P. **JUUTINEN**)

Abstract:

I will consider the non-linear optimal transportation problem of minimizing the cost functional:

$C_\infty(\lambda) = \inf_{\gamma} \int_{\Omega^2} |y-x| d\gamma$

in the set of probability measures on Ω^2 having prescribed marginals.

This corresponds to the question of characterizing the measures that realize the infinite Wasserstein distance. I will establish the existence of "local" solutions and characterize this class with aid of an adequate version of cyclical monotonicity. Moreover, under natural assumptions, I will show that local solutions are induced by transport maps.

The lack of duality forces us to introduce a different technique to prove the existence of an optimal transport map. At the end of the talk I will show how this technique may be useful to tackle the classical Monge problem for general norms.

(Joint paper with T.Champion and P.Juutinen, work in progress with T. Champion)

Thanks:

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